Lecture 32

10.1 - Parametric Equations

Our goal for this section is to represent planar curves using one variable to describe how the x- & y-coordinates of the curve change.

Def: Let $x = f(t)$ and $y = g(t)$, and let I be an interval. The set of points $(x,y) = (f(t), g(t))$ for t in I is called a parametric curve and $x = f(t)$ & $y = g(t)$ are called the parametric equations of the curve. If $I = [a,b]$, $(f(a), g(a))$ is called the initial point & $(f(b), g(b))$ is the terminal point.

The idea of parametric equations is that a curve is "1-dimensional" so we should be able to describe it with only one variable (locally, at least). In Calc III, you'll see this idea again, as well as for surfaces!

Ex: Sketch the parametric curve

$$x = \cos t, \quad y = \sin t, \quad 0 \le t \le 2\pi.$$

What is the curve?

Ex: Sketch the parametric curve

$$x = \sin 2t, \quad y = \cos 2t, \quad 0 \le t \le 2\pi$$

What is the curve?

Q: How is the second curve different from the first?

Def: The direction in which you travel along a curve is called the orientation of the curve. (You travel with increasing t-values.)

Ex: Sketch the parametric curve

$$x=\sqrt{t}, \ y=1-t, \ 0 \le t < \infty.$$

What is the orientation of the curve?

Sometimes, changing back to Cartesian coordinates is useful to identify the curve:

Ex: Describe the curve in the last example in Cartesian coordinates:

Ex: Convert the curve given by

$$x = 3\sin t,\ y = 2\cos t,\ \frac{\pi}{2} \le t \le \frac{9\pi}{2}$$

to Cartesian coordinates and describe the curve as t increases.

We can also change from Cartesian to parametric.

Easiest Case: Let x or $y = t$, then solve for the other.

Ex: Give parametric equations for $f(x) = x^3$.

Ex: Give parametric equations for $y^2 = x^3$.

Sometimes we have to rely on other known equations:

Ex: Give parametric equations describing the motion of a particle which travels around the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ counterclockwise once, starting at $(-a, 0)$.